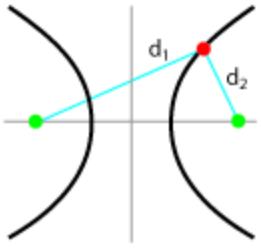
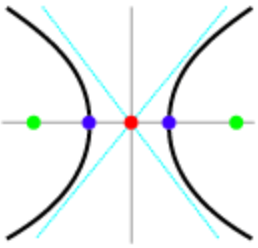
Conic Sections: Hyperbolas

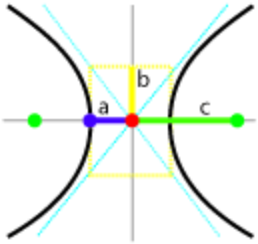
A hyperbola is the set of all points where the difference of the distance from 2 given points (foci) and the curve is constant.

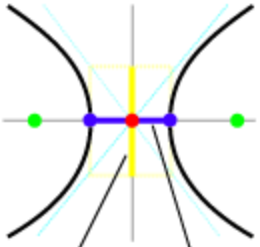
ie, $d_1 - d_2 =$ a constant



A curve with this equation: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ is a hyperbola that opens left and right.







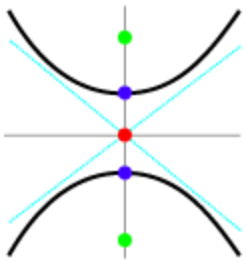
- -- the center = (h, k)
- -- a focus (there are 2) = $(h \pm c, k)$
- -- vertices = $(h \pm a, k)$
- -- asymptotes $y = \pm \frac{b}{a}(x-h) + k$

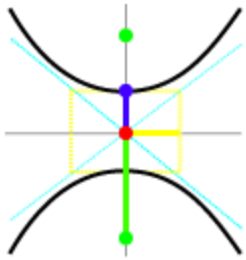
$c^2 = a^2 + b^2$

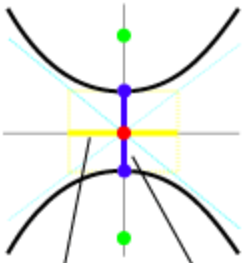
transverse axis = 2a

conjugate axis = 2b

A curve with this equation: $\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$ is a hyperbola that opens up and down.





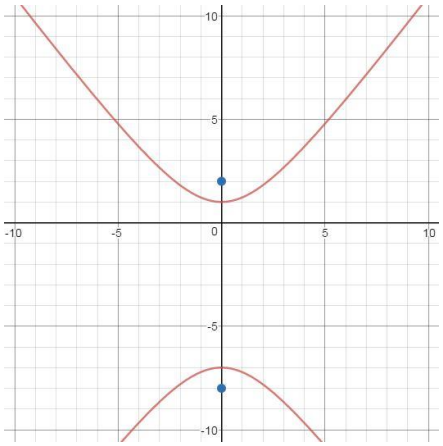


- -- the center = (h, k)
- -- a focus (there are 2) = $(h, k \pm c)$
- -- vertices = $(h, k \pm a)$
- -- asymptotes $y = \pm \frac{b}{a}(x-h) + k$

$c^2 = a^2 + b^2$

transverse axis = 2b

conjugate axis = 2b



Ex: What is the equation of the hyperbola: → it's vertical

$center = (-3, 0) \rightarrow h = -3,\ k = 0 \qquad a = 4,\ c = 5$

$c^2 - a^2 = b^2 = 25 - 16 = 9 \rightarrow b = 3$ So $\frac{(y-0)^2}{16} - \frac{(x+3)^2}{9} = 1$

Ex: For the hyperbola: $\frac{(x-5)^2}{4} - \frac{(y+4)^2}{36} = 1$, what are the coordinates of the foci? Horizontal so the foci are at $(h \pm c, k)$.

$(h, k) = (5, -4) \rightarrow h = 5, \qquad k = -4$

$a^2 = 4, b^2 = 36 \rightarrow c^2 = a^2 + b^2 = 4 + 36 = 40 \rightarrow c = \sqrt{40} \rightarrow foci = (5 \pm \sqrt{40}, -4)\ or\ (5 \pm 2\sqrt{10}, -4)$